ASYMPTOTIC ANALYSIS OF STATIONARY DISTRIBUTION OF THE FRONT OF A TWO-STAGE CONSECUTIVE EXOTHERMIC REACTION IN A CONDENSED MEDIUM

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An approximate theory of the stationary distribution of the plane front of a two-stage exothermic consecutive chemical reaction in a condensed medium is developed in the article. The method of joined asymptotic expansions is used in constructing the solutions. The ratio of the sum of the activation energies of the reactions to the final adiabatic combustion temperature is a parameter of the expansion. The characteristic limiting states of the stationary distribution of the wave corresponding to different values of the parameters figuring in the problem are shown. Approximate analytical expressions for the wave velocity and distribution of concentrations are obtained for each of the states.

1. Formulation of the Problem. The stationary distribution of the plane front of a two-stage consecutive exothermal reaction $A_1 \rightarrow A_2 \rightarrow A_3$ in a condensed medium can be described by the following system of equations and boundary conditions:

$$\frac{d}{dx}\left(\lambda \frac{dT}{dx}\right) - mc\frac{dT}{dx} + Q_1 a_1 \rho \Phi_1(T) + Q_2 a_2 \rho \Phi_2(T) = 0 \tag{1.1}$$

$$m\frac{da_1}{dx} = -a_1 \rho \Phi_1(T) \tag{1.2}$$

$$m\frac{da_2}{dx} = a_1 \rho \Phi_1(T) - a_2 \rho \Phi_2(T) \tag{1.3}$$

$$\Phi_1(T) = k_1 \exp\frac{-E_1}{RT}, \quad \Phi_2(T) = k_2 \exp\frac{-E_2}{RT} \tag{1.4}$$

$$x = -\infty, \quad a_1 = 1, \quad T = T_-, \quad a_2 = 0 \tag{1.5}$$

$$x = \infty, \quad a_1 = a_2 = 0, \quad T = T_+ = T_- + c^{-1}(Q_1 + Q_2)\ . \tag{1.6}$$

Here x is the coordinate, a_1 and a_2 are the mass fractions of the substances A_1 and A_2, T is the temperature, ρ is the density, m is the mass combustion rate, c is the heat capacity, λ is the thermal conductivity, R is the gas constant, Q_1 and Q_2 are the thermal effects of the reactions, k_1 and k_2 are pre-exponential multipliers, and E_1 and E_2 are the activation energies.

It is assumed that in the course of the chemical reactions the density and all the thermophysical characteristics of the medium maintain constant values and that the rates of the chemical reactions depend on the temperature according to the Arrhenius law.

The problem (1.1)-(1.6) is a two-point boundary problem whose solution consists of the definite functions $a_1(x)$, $a_2(x)$, and T(x) and the proper value m of the problem.

For the existence of a solution it is assumed that the function Φ_1 is different from zero and is determined by Eq. (1.4) everywhere except for the small temperature interval $T_- \leq T < T_\varepsilon$, where it is reduced to zero [1, 2].

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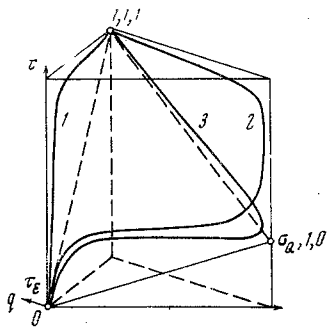

Fig. 1

The problem (1.1)-(1.6) has the initial integral

$$\lambda dT/dx = mc\,(T - T_+) + m\,(Q_1 + Q_2)\,a_1 + mQ_2 a_2 \;. \qquad (1.7)$$

Equation (1.7) will be used in place of Eq. (1.1). If the temperature is taken as an independent variable, the problem (1.2)-(1.7) can be represented in the form

$$\mu \frac{dr}{d\tau} = \frac{\sigma_k(1-r)}{\tau - \sigma_Q r - (1-\sigma_Q)\,q}\exp\left[-\beta\sigma_E - \frac{\beta\sigma_E(1-\tau)}{\sigma+\tau}\right] \qquad (1.8)$$

$$\mu \frac{dq}{d\tau} = \frac{(1-\sigma_k)(r-q)}{\tau - \sigma_Q r - (1-\sigma_Q)\,q}\exp\left[-\beta(1-\sigma_E) - \frac{\beta(1-\sigma_E)(1-\tau)}{\sigma+\tau}\right] \qquad (1.9)$$

$$\tau = 0, \qquad r = 0, \qquad q = 0 \qquad (1.10)$$

$$\tau = 1, \qquad r = 1, \quad q = 1 \qquad (1.11)$$

$$r = 1 - a_1, \quad q = 1 - a_1 - a_2, \quad \sigma_k = \frac{k_1}{k_1 + k_2}, \quad \sigma_E = \frac{E_1}{E_1 + E_2}$$

$$\sigma_Q = \frac{Q_1}{Q_1 + Q_2}, \quad \beta = \frac{E_1 + E_2}{RT_+}, \quad \tau = \frac{T - T_-}{T_+ - T_-}, \quad \sigma = \frac{T_-}{T_+ - T_-}, \quad \mu = \frac{m^2 c}{\lambda\rho\,(k_1 + k_2)} \;.$$

Here $r(\tau)$ and $q(\tau)$ are the functions sought and μ is a proper value of the problem.

2. Some Properties of the Integral Curves. In the space τ, r, q the integral curves (1.8)-(1.11) of the problem must pass through the points (τ_ε, 0, 0) and (1, 1, 1), $0 < \tau_\varepsilon \ll 1$. From the conditions of nonnegativity of the concentration and successiveness of the conversion $r \geq q \geq 0$ of the reagents, and the condition of nonnegativity of the temperature gradient (heat flux) $\tau - \sigma_Q r - (1 - \sigma_Q)q \geq 0$, as well as from the boundary conditions,it can be concluded that the region where the integral curves have physical meaning is bounded by the five planes (see Fig. 1)

$$q = 0, \qquad r = q, \qquad \tau = 1, \qquad r = 1,$$
$$\tau - \sigma_Q r - (1 - \sigma_Q)\,q = 0 \;.$$

The point (τ_ε, 0, 0), corresponding to the cold boundary of the combustion wave, is an ordinary point, while the point (1, 1, 1), corresponding to the hot boundary of the combustion zone, is singular. The following three integral curves pass through it:

$$\left(\frac{dr}{d\tau}\right)_1 = \left(\frac{dq}{d\tau}\right)_1 = 0 \qquad (2.1)$$

$$\left(\frac{dr}{d\tau}\right)_2 = 0, \quad \left(\frac{dq}{d\tau}\right)_2 = \frac{1}{1-\sigma_Q} + \frac{1-\sigma_k}{\mu(1-\sigma_Q)}\,e^{-\beta(1-\sigma_E)} \qquad (2.2)$$

$$\left(\frac{dr}{d\tau}\right)_3 = -\left(\frac{dq}{d\tau}\right)_3\left[\frac{\sigma_k}{1-\sigma_k}\,e^{(1-2\sigma_E)\beta} - 1\right],$$

$$\left(\frac{dq}{d\tau}\right)_3 = \left(1 + \frac{\sigma_k}{\mu}\,e^{-\beta\sigma_E}\right)\left[1 - \frac{\sigma_Q\sigma_k}{1-\sigma_k}\,e^{(1-2\sigma_E)\beta}\right]^{-1} . \qquad (2.3)$$

The first of these curves could not be a solution of the problem for any values of the parameters since this curve does not fall within the region of admissible values of τ, r, and q (Fig. 1). The other two curves can represent a solution. It is seen from (2.3) that for large β one of the derivatives must be negative if $\sigma_E < 1/2$. Such a curve cannot be a solution since the functions r and q must be increasing. For $\sigma_E < 1/2$ the solution of the problem is represented by the curve (2.2).

Let us note one more property of the equations (1.8), (1.11). The straight line $\tau - \sigma_Q r - (1 - \sigma_Q)q = 0$ on the $r = 1$ plane between the points (σ_Q, 1, 0) and (1, 1, 1) consists of the singular points of Eq. (1.8). Each of these is a saddle point through which two separatrices pass in the plane q = const which have the tangents

$$\left(\frac{dr}{d\tau}\right)^{(1)} = 0, \qquad \left(\frac{dr}{d\tau}\right)^{(2)} = \frac{\sigma_k}{\mu\sigma_Q}\exp\frac{-\beta\sigma_E(1+\sigma)}{\sigma_Q+\sigma} - \frac{1}{\sigma_Q} \;. \qquad (2.4)$$

3. Preliminary Analysis of Equations. Description of Particular Cases. Approximate solutions of the problem (1.8)-(1.11), corresponding to different values of the parameter σ_E, will be constructed in parts 4-7 by the method of joined asymptotic expansions [3]. These particular cases are developed in a successive analysis of different assumptions concerning the possible asymptotic behavior of the proper value μ and the functions r and q for large β. Before going on to the construction of the solutions, let us note the line of reasoning uniquely leading to the various important particular cases studied further.

It follows from the form of Eqs. (1.8), (1.9) that for large values of β the region in which the principal variation in the functions r and q occurs can occupy a small fraction of the interval $0 \leq \tau \leq 1$. The right sides of Eqs. (1.8) and (1.9) contain small exponential multipliers which must be compensated for at large values of β in order for the functions r and q to grow from 0 to 1 in the interval $0 \leq \tau \leq 1$. In order to compensate for these multipliers, it is necessary to assume that the quantity μ contains the multiplier $\exp(-\beta\alpha)$, so that compensation of the exponential terms in Eqs. (1.8) and (1.9) will occur at the points $\tau = \tau_1°$ and $\tau_2°$:

$$\tau_1° = \alpha^{-1}\sigma_E(1+\sigma) - \sigma, \qquad \tau_2° = \alpha^{-1}(1-\sigma_E)(1+\sigma) - \sigma \,. \tag{3.1}$$

The choice of the constant α determines the position of the points $\tau_1°$ and $\tau_2°$ and the asymptotic behavior of the functions $r(\tau)$ and $q(\tau)$.

Let $0 \leq \tau_1° \leq 1$, $\tau_2° < 0$. For such a choice of α the exponential multiplier is compensated for only in the expression for the derivative $dr/d\tau$, while in the expression for $dq/d\tau$ an increasing exponent appears. For $\tau < \tau_1°$ the derivative $dr/d\tau$ and the function $r(\tau)$ are exponentially small. The small neighborhood of the point $\tau = \tau_1°$ is the region of the principal variation in the function $r(\tau)$ from zero to one. The compensation for the growing exponential multiplier in the expression for $dq/d\tau$ can be provided for by the corresponding behavior of the difference $r - q$ which must be exponentially small, so that the region of the principal variation of q from 0 to 1 will coincide with the corresponding region of variation of r, i.e., it will be represented by the small neighborhood of the point $\tau = \tau_1°$. It then follows from the condition $\tau - \sigma_Q\tau - (1-\sigma_Q)q > 0$ that this is possible if $\tau_1° = 1$. From (3.1) we find that $\alpha = \sigma_E$. Having divided (1.9) by (1.8), we obtain

$$\frac{dq}{dr} = \frac{r-q}{1-r}\, e^{-\beta(1-2\sigma_E)} \exp\frac{-\beta(1-2\sigma_E)(1-\tau)}{\sigma+\tau} \,. \tag{3.2}$$

It is seen from Eq. (3.2) that the region of the principal variation of q can coincide with the region of the principal variation of r only when $\sigma_E > ½$ and must be located in the small neighborhood of $\tau = 1$. The case $\sigma_E = ½$ will differ somewhat from the case $½ < \sigma_E \leq 1$. Although the regions of the principal variation of r and q will coincide here and be located near $\tau = 1$, the difference $r - q$ will still not be exponentially small.

The integral curves (1.8)-(1.11) of the problem for $½ < \sigma_E \leq 1$ with large β are characterized by the same asymptotic behavior. At the singular point (1, 1, 1) they correspond to the values of the derivatives determined by Eq. (2.3). The behavior of these curves which pass close to the straight lines $r = 0$, $q = 0$ and $r = q$, $r = 1$ is shown in Fig. 1 (curve 1).

Now suppose the points $\tau = \tau_1°$ and $\tau = \tau_2°$ lie within the interval $0 \leq \tau \leq 1$. In this case, as before, the principal variation of r will occur in the neighborhood of $\tau = \tau_1°$; for $\tau > \tau_1°$ the derivative $dr/d\tau$ again becomes small, since in this case the growth of the exponential multiplier in the expression for $dr/d\tau$ is expressed even more sharply than the decrease in $(1 - r)$. The principal variation of q from 0 to 1 will occur near $\tau = \tau_2°$, with the exponential multiplier in the expression for the derivative $dq/d\tau$ reducing to unity. It follows from the condition $r > q$ that $\tau_1° < \tau_2°$, while from the condition $\tau - \sigma_Q r - (1-\sigma_Q)q > 1$ it follows that $\tau_1° > \sigma_Q$, $\tau_2° = 1$. Taking (3.1) into account, we find

$$\alpha = 1 - \sigma_E, \qquad (\sigma_Q + \sigma)(1 + \sigma_Q + 2\sigma)^{-1} < \sigma_E < 1/2 \,.$$

In this case the integral curves emerging from the point $(\tau_\varepsilon, 0, 0)$ travel along the line $r = 0$, $q = 0$, then staying near the surface $q = 0$, they move to the line $r = 1$, $q = 0$, and after turning near the point (1, 1, 0) move in the vicinity of the line $r = \tau = 1$ to the singular point (1, 1, 1) (curve 2 in Fig. 1). The derivatives of the integral curves at the singular point are determined by Eq. (2.2), from which it can be found that in the case under consideration $(dq/d\tau)_+ \to \infty$.

For $0 < \sigma_E < (\sigma_Q + \sigma)(1 + \sigma_Q + 2\sigma)^{-1}$ the integral curves (1.8)-(1.11) of the problem with large β also possess general characteristics. In this case one must choose $\alpha = \sigma_E(1+\sigma)(\sigma_Q+\sigma)^{-1}$. Here $\tau_1° = \sigma_Q$ and $\tau_2° > 0$; the exponential multiplier $\exp(-\alpha\beta)$ compensates only for the small exponential multiplier in the expression for $dr/d\tau$ while the "equalization" of orders of magnitude in the left and right sides of Eq. (1.9) is provided for by the corresponding exponential behavior of the denominator in the right side of (1.9). The growth of r from 0 to 1 occurs in the small neighborhood of the point $\tau \approx \sigma_Q$. The increase of q from 0 to 1 occurs "uniformly" over the entire section $\sigma_Q \leq \tau \leq 1$. The behavior of the integral curve is shown in Fig. 1 (curve 3). At the singular point (1, 1, 1) the integral curves have the derivatives

(2.2). One can ascertain that in contrast to curve 2 the derivative $dq/d\tau$ has a finite value at the singular point in this case.

It can be concluded from the qualitative analysis conducted that the asymptotic behavior of the integral curves of the problem differ considerably with variation in the value of σ_E in the following intervals:

$$1/_2 < \sigma_E \leqslant 1, \qquad \sigma_E = 1/_2, \qquad (\sigma_Q + \sigma)(1 + \sigma_Q + 2\sigma)^{-1} < \sigma_E < 1/_2$$
$$0 \leqslant \sigma_E < (\sigma_Q + \sigma)(1 + \sigma_Q + 2\sigma)^{-1}.$$

This conclusion is confirmed by the actual construction of four different solutions. Knowledge of the asymptotic solutions of the simpler problem presented in [4] proves to be useful in choosing the form of the asymptotic expansion.

4. Solution for $\frac{1}{2} < \sigma_E \leq 1$. Two regions having different asymptotic behavior of the solutions must be distinguished in this case: the small neighborhood of the point $\tau = 1$ (inner region) and the remaining part of the interval (outer region). In the inner region in place of τ we introduce the variable $\tau^* = \beta(1-\tau)$, and we will seek a solution in each of the regions in the form of inner and outer expansions

$$r(\tau^*) = f_0(\beta) r_0(\tau^*) + f_1(\beta) r_1(\tau^*) + \ldots, \quad r(\tau) = F_0(\beta) r_0(\tau) + F_1(\beta) r_1(\tau) + \ldots$$
$$(f_1/f_0 \to 0, \ F_1/F_0 \to 0, \ \beta \to \infty) \tag{4.1}$$

$$q(\tau^*) = n_0(\beta) q_0(\tau^*) + n_1(\beta) q_1(\tau^*) + \ldots, \ q(\tau) = N_0(\beta) q_0(\tau) + N_1(\beta) q_1(\tau) + \ldots$$
$$(n_1/n_0 \to 0, \ N_1/N_0 \to 0, \ \beta \to \infty) \cdot \tag{4.2}$$

In the two regions we seek an expansion for the proper value μ in the form

$$\mu = \varphi_0(\beta) \mu_0 + \varphi_1(\beta) \mu_1 + \ldots, \quad \varphi_1/\varphi_0 \to 0 \ \text{ as } \ \beta \to \infty. \tag{4.3}$$

The outer expansion must satisfy the boundary conditions (1.10), and the inner expansion must satisfy the conditions (1.11). The correspondence between the expansions in the outer and inner regions is established from the condition of joining, which consists in the requirement of the identical limiting behavior of the inner and outer expansions written in identical variables [3, 4]. We will confine ourselves to the determination of two terms of the expansions (4.1)-(4.3).

After converting to the variable τ^* and substituting the expansions (4.1)-(4.3) into (1.8), (1.9), and (1.11) with the accuracy being approximately equal to the higher-order terms in smallness, we obtain

$$-\beta\varphi_0\mu_0 \frac{dr_0}{d\tau^*} = \frac{\sigma_k(1-r_0)}{1-\sigma_Q r_0 - (1-\sigma_Q) q_0} \exp\left[-\beta\sigma_E - \frac{\sigma_E \tau^*}{1+\sigma}\right] \tag{4.4}$$

$$-\beta\varphi_0\mu_0 \frac{dq_0}{d\tau^*} = \frac{(1-\sigma_k)(r_0 + f_1 r_1 - q_0 - n_1 q_1)}{1-\sigma_Q r_0 - (1-\sigma_Q) q_0} \exp\left[-\beta(1-\sigma_E) - \frac{(1-\sigma_E)\tau^*}{1+\sigma}\right] \tag{4.5}$$

$$\tau^* = 0, \qquad r_0 = 1, \ q_0 = 1. \tag{4.6}$$

The equality $n_0(\beta) = f_0(\beta) = 1$, which follows from (4.6), is used in Eqs. (4.4) and (4.5).

It is seen from (4.4) that to equalize the orders of smallness of the left and right sides in (4.4) one must choose

$$\varphi_0(\beta) = \beta^{-1} \exp(-\beta\sigma_E). \tag{4.7}$$

In this case Eq. (4.5) can be satisfied if one takes

$$r_0 = q_0, \qquad f_1 = n_1, \qquad r_1 = q_1. \tag{4.8}$$

This means that the difference between the functions r_0, q_0 and r_1, q_1 is exponentially small:

$$r_0 + f_1 r_1 - q_0 - n_1 q_1 \sim \frac{\sigma_k}{1-\sigma_k} e^{-(2\sigma_E - 1)\beta} \exp\frac{-(2\sigma_E - 1)\tau^*}{1+\sigma}. \tag{4.9}$$

Taking (4.7) and (4.8) into account, we now find from (4.4) and (4.6) that

$$r_0(\tau^*) = 1 - \frac{\sigma_k(1+\sigma)}{\mu_0\sigma_E}\left[1 - \exp\frac{-\sigma_E\tau^*}{1+\sigma}\right]. \tag{4.10}$$

Keeping (4.7) in mind one can confirm that the solutions $r(\tau)$ and $q(\tau)$ of Eqs. (1.8) and (1.9) are exponentially small in the outer region. Therefore, in the outer expansions (4.1) and (4.2)

$$r_0(\tau) = q_0(\tau) = r_1(\tau) = q_1(\tau) = 0 .$$

The condition of joining then leads to the requirement that

$$r_0(\tau^*) \to 0, \quad q_0(\tau^*) \to 0, \quad r_1(\tau^*) \to 0, \quad q_1(\tau^*) \to 0 \text{ as } \tau^* \to \infty . \tag{4.11}$$

Applying (4.11) to (4.10), we find

$$\mu_0 = \frac{\sigma_k(1+\sigma)}{\sigma_E}, \qquad r_0(\tau^*) = \exp\frac{-\sigma_E\tau^*}{1+\sigma} . \tag{4.12}$$

Let us proceed to the determination of the second approximation. After converting to the variable τ^* and substituting the two-term expansions (4.1)-(4.3) into (1.8) and (1.9) with allowance for (4.9), retaining in the equations terms of a higher order of smallness than during the determination of r_0, q_0, and μ_0, we obtain for $r_1(\tau^*)$ the equation

$$\beta\mu_1\varphi_1\frac{dr_0}{d\tau^*} + \beta\mu_0\varphi_0 f_1\frac{dr_1}{d\tau^*} = \sigma_k\frac{1}{\beta}\left[\frac{\sigma_E\tau^{*2}}{(1+\sigma)^2} - \frac{\tau^*}{1-r_0}\right]\exp\left(-\beta\sigma_E - \frac{\sigma_E\tau^*}{1+\sigma}\right) . \tag{4.13}$$

It becomes possible to determine the function $r_1(\tau^*)$, satisfying the boundary condition and the joining condition, from (4.13) if

$$f_1(\beta) = \beta^{-1}, \quad \varphi_1(\beta) = \beta^{-1}\varphi_0(\beta) = \beta^{-2}\exp(-\beta\sigma_E) . \tag{4.14}$$

The equation and the boundary condition for r_1 take the form

$$\mu_0\frac{dr_1}{d\tau^*} = \sigma_k\left[\frac{\sigma_E\tau^{*2}}{(1+\sigma)^2} - \frac{\tau^*}{1-r_0(\tau^*)} + \frac{\mu_1}{\mu_0}\right]\exp\frac{-\sigma_E\tau^*}{1+\sigma} \tag{4.15}$$
$$\tau^* = r_1 = 0.$$

Here μ_0 and $r_0(\tau^*)$ are determined by Eqs. (4.12). From (4.15) we find

$$r_1(\tau^*) = \frac{\mu_1\sigma_E}{\sigma_k(1+\sigma)}\left[1 - \exp\frac{-\sigma_E\tau^*}{1+\sigma}\right] + \frac{2}{\sigma_E} - \frac{(1+\sigma)}{\sigma_E}J\left(\frac{\sigma_E\tau^*}{1+\sigma}\right) - \frac{2}{\sigma_E}\left[1 + \frac{\sigma_E\tau^*}{1+\sigma} + \frac{\sigma_E^2\tau^{*2}}{2(1+\sigma)}\right]\exp\frac{-\sigma_E\tau^*}{1+\sigma} \tag{4.16}$$

$$J(\alpha) = \int_0^\alpha (e^z - 1)^{-1} z\,dz, \quad J(\infty) = \frac{\pi^2}{6} .$$

Applying the joining condition (4.11) to (4.16), we find

$$\mu_1 = \frac{\sigma_k(1+\sigma)}{\sigma_E^2}\left[(1+\sigma)\frac{\pi^2}{6} - 2\right] . \tag{4.17}$$

Thus, the two-term inner and outer expansions of the functions $r(\tau)$ and $q(\tau)$ and the expansion of the proper value μ, which represent an approximate solution of the problem (1.8)-(1.11) in the particular case under consideration, have the form

$$r(\tau^*) = q(\tau^*) = r_0(\tau^*) + \beta^{-1}r_1(\tau^*), \qquad r(\tau) = q(\tau) = 0, \tag{4.18}$$
$$\mu = \beta^{-1}(\mu_0 + \beta^{-1}\mu_1)\exp(-\beta\sigma_E).$$

Here r_1, r_0, μ_0, and μ_1 are determined by the equations (4.12), (4.16), and (4.17). Let us write the two-term expansion for the mass rate of propagation of combustion in dimensional variables:

$$m = \left[\frac{k_1\lambda\rho(1+\sigma)}{c}\right]^{1/2}\left(\frac{E_1}{RT_+}\right)^{-1/2}\left\{1 + \left(\frac{E_1}{RT_+}\right)^{-1}\left[(1+\sigma)\frac{\pi^2}{12} - 1\right]\right\}\exp\frac{-E_1}{2RT_+} . \tag{4.19}$$

It is seen that in the particular case under consideration the combustion rate depends only on the kinetic parameters k_1 and E_1 of the initial reaction.

Comparing (4.19) with the equation obtained in [4] for the rate of propagation of a single-stage exothermal reaction in the condensed phase shows that the start of the second reaction occurs only at a value of the combustion temperature which is determined by the total heat release of the two reactions.

5. Solution for $\sigma_E = 1/2$. The system of equations (1.8), (1.9) can be reduced to a single equation. After substituting $\sigma_E = 1/2$ and dividing (1.9) by (1.8), we obtain

$$\frac{dq}{dr} = \frac{(1-\sigma_k)}{\sigma_k}\,\frac{r-q}{1-r} . \tag{5.1}$$

With the help of (5.1) we can express q through r:

$$q(r)=\begin{cases}(1-\delta_k)^{-1}[1-\delta_k r-(1-r)^{\delta_k}], & \delta_k=\sigma_k^{-1}(1-\sigma_k),\ \sigma_k\neq 1/2\\ r+(1-r)\ln(1-r), & \sigma_k=1/2\end{cases} \tag{5.2}$$

Making use of (5.2), the problem (1.8)-(1.11) reduces to the solution of the single equation with the conditions

$$\mu\frac{dr}{d\tau}=\frac{\sigma_k(1-r)}{\tau-\sigma_Q r-(1-\sigma_Q)q(r)}\exp\left[-\frac{\beta}{2}-\frac{\beta(1-\tau)}{2(\sigma+\tau)}\right] \tag{5.3}$$

$$\tau=0,\qquad r=0 \tag{5.4}$$

$$\tau=1,\qquad r=1. \tag{5.5}$$

Here the function q(r) is given by Eq. (5.2).

In constructing the approximate solution of the problem (5.3)-(5.5) we distinguish inner and outer regions of variation in the variable τ as in part 4; in the neighborhood of $\tau=1$ we introduce the variable $\tau^*=\beta(1-\tau)$ and we will seek a function r and a proper value μ in the form of the expansions (4.1)-(4.3). For the null approximation we obtain from (5.3) and (5.5)

$$-\mu_0\beta\varphi_0\frac{dr_0}{d\tau^*}=\frac{\sigma_k(1-r_0)}{1-\sigma_Q r_0-(1-\sigma_Q)q_0(r_0)}\exp\left[-\frac{\beta}{2}-\frac{\tau^*}{2(1+\sigma)}\right],\qquad r_0(0)=1. \tag{5.6}$$

Here, as in part 4, $f_0(\beta)=1$. The dependence $q_0(r_0)$ is established from (5.2). To compensate for the small exponential term in the right side of Eq. (5.6), we set

$$\varphi_0(\beta)=\beta^{-1}\exp(-\beta/2). \tag{5.7}$$

Using (5.7) and (5.2), one can find from (5.6) that

$$\mu_0(1-r_0)\{1+(1-\sigma_Q)[1-\ln(1-r_0)]\}=(1+\sigma)\left[1-\exp\frac{-\tau^*}{2(1+\sigma)}\right] \tag{5.8}$$
$$(\sigma_k=1/2)$$

$$\frac{\mu_0(1-r_0)}{1-\delta_k}\left[\sigma_Q-\delta_k+\frac{1-\sigma_Q}{\delta_k}(1-r_0)^{\delta_k-1}\right]=2\sigma_k(1+\sigma)\left[1-\exp\frac{-\sigma_E\tau^*}{1+\sigma}\right]. \tag{5.9}$$
$$(\sigma_k\neq 1/2)$$

Applying the condition of joining, which as in part 4 is expressed through the requirement that $r_0(\tau^*)\to 0$ as $\tau^*\to\infty$ to (5.8) and (5.9), we obtain

$$\mu_0=\frac{(1-\sigma_k)\sigma_k(1+\sigma)}{(1-\sigma_k\sigma_Q)\sigma_E},\quad m_0=\left[\frac{k_1k_2\lambda\rho T_+}{k_1Q_2+k_2(Q_1+Q_2)}\right]^{1/2}\left(\frac{E}{RT_+}\right)^{-1/2}\exp\frac{-E}{2RT_+}. \tag{5.10}$$

Equations (5.8)-(5.10) determine the null approximation for the function and the proper value μ for the problem. The following terms of the expansions, which provide a correction on the order of β^{-1}, cannot be obtained in analytical form, and their determination comes down to the numerical integration of a simple ordinary first-order differential equation:

$$\frac{dr_1}{d\tau^*}=\left\{\frac{\tau^{*2}}{2(1+\sigma)^2}-\frac{\tau^*+\sigma_Q r_1+r_1(1-\sigma_Q)\delta_k[r_0-q_0(r_0)](1-r_0)^{-1}}{1-\sigma_Q r_0-(1-\sigma_Q)q_0(r_0)}+\frac{r_1}{1-r_0}+\right.$$
$$\left.+\frac{\mu_1}{\mu_0}\right\}\frac{\sigma_k}{\mu_0}\frac{(1-r_0)}{[1-\sigma_Q r_0-(1-\sigma_Q)q_0(r_0)]}\exp\frac{-\tau^*}{2(1+\sigma)},\quad r_1(0)=0. \tag{5.11}$$

We note that the expression (5.10) for μ_0 coincides with (4.12) with $\sigma_k\ll 1$. This result reflects the fact that with the equality of the activation energies of the two reactions the relative magnitude of the pre-exponential multipliers becomes the determining factor in the comparison of the chemical reaction rates. For $k_2\gg k_1$ the difference $r-q$ is small, and in contrast to part 4 it becomes insignificant.

6. Solution for $(\sigma_Q+\sigma)(1+\sigma_Q+2\sigma)^{-1}<\sigma_E<1/2$. In this case the division of the interval $0\le\tau\le 1$ into inner and outer regions will be different for Eqs. (1.8) and (1.9). For Eq. (1.9) the inner region will be the small neighborhood of $\tau=\tau_2^\circ=1$, the outer region will be the remaining part of the interval. For Eq. (1.8) the inner region will be the small neighborhood of some point $\tau=\tau_1^\circ<1$, and the outer region will consist of two segments of the interval $0\le\tau\le 1$ separating the small neighborhood of the point τ_1° from the points $\tau=0$ and $\tau=1$. The construction of the solution comes down to the search for the outer and inner expansions for each of the two partitionings of the interval.

Let us study the region adjacent to $\tau = \tau_2^\circ = 1$. In place of the variable τ in (1.8)-(1.11) we introduce $\tau_2^* = \beta(1-\tau)$. We will seek an approximate solution in the inner region in the form of the two-term expansions

$$\mu = \varphi_0(\beta)\mu_0 + \varphi_1(\beta)\mu_1, \qquad r(\tau_2^*) = f_0(\beta) r_0(\tau_2^*) + f_1(\beta) r_1(\tau_2^*)$$
$$q(\tau_2^*) = n_0(\beta) q_0(\tau_2^*) + \eta_1(\beta) q_1(\tau_2^*) \tag{6.1}$$
$$\varphi_1/\varphi_0 \to 0, \qquad f_1/f_0 \to 0, \qquad n_1/n_0 \to 0 \quad \text{as} \quad \beta \to \infty.$$

From the boundary conditions (1.11) we find

$$f_0(\beta) = n_0(\beta) = 1 \ . \tag{6.2}$$

In the particular case under consideration one must choose

$$\varphi_0(\beta) = \beta^{-1} \exp[-(1-\sigma_E)\beta], \quad \varphi_1(\beta) = \beta^{-1}\varphi_0(\beta),$$
$$n_0(\beta) = 1, \ n_1(\beta) = \beta^{-1} \ . \tag{6.3}$$

After substituting (6.1) into Eq. (1.8) and taking (6.2) and (6.3) into account, one can obtain

$$1 - r(\tau_2^*) \sim \exp(2\sigma_E - 1)\beta \ .$$

It is seen that the function $r(\tau_2^*)$ as $\beta \to \infty$ differs from unity only in a small exponential term, so that

$$r_0(\tau_2^*) = 1, \qquad r_1(\tau_2^*) = 0 \ .$$

The equation and boundary conditions for the function $q_0(\tau_2^*)$ have the form

$$-\mu_0 \frac{dq_0}{d\tau_2^*} = \frac{(1-\sigma_k)}{(1-\sigma_Q)} \exp\frac{-(1-\sigma_E)\tau^*}{1+\sigma}, \quad q_0(0) = 1 \ . \tag{6.4}$$

From (6.4) we find

$$q_0(\tau_2^*) = 1 - \frac{(1-\sigma_k)(1+\sigma)}{\mu_0(1-\sigma_Q)(1-\sigma_E)}\left[1 - \exp\frac{-(1-\sigma_E)\tau_2^*}{1+\sigma}\right] \ . \tag{6.5}$$

The constant μ_0 in (6.5) must be determined from the condition of joining (6.5) with the solution $q(\tau)$ in the outer region, which is equal to zero with the accuracy of the exponential terms. Therefore, the condition of joining is expressed by the requirement that

$$q_0(\tau_2^*) \to 0, \qquad q_1(\tau_2^*) \to 0 \quad \text{as} \quad \tau^*_2 \to \infty \ . \tag{6.6}$$

Applying (6.6) to (6.5), we obtain

$$\mu_0 = \frac{(1-\sigma_Q)(1-\sigma_E)}{(1-\sigma_k)(1+\sigma)}, \quad q_0(\tau_2^*) = \exp\frac{-(1-\sigma_E)\tau_2^*}{1+\sigma} \ . \tag{6.7}$$

After substituting $\tau_2^* = \beta(1-\tau)$ into (6.1), having retained terms with a higher order of smallness, taking (6.2)-(6.4) into account, one can obtain from (1.9) an equation for q_1:

$$\mu_0 \frac{dq_1}{d\tau_2^*} = \frac{(1-\sigma_k)}{(1-\sigma_Q)}\left\{\frac{\mu_1}{\mu_0} + \frac{(1-\sigma_E)\tau_2^{*2}}{(1+\sigma)^2} - \frac{\tau_2^*}{(1-\sigma_Q)(1-q_0)}\right\}\exp\frac{-(1-\sigma_E)\tau_2^*}{1+\sigma} \ . \tag{6.8}$$

The solution of Eq. (6.8) satisfying the condition $q_1(0) = 0$ has the form

$$q_1(\tau_2^*) = \frac{\mu_1(1-\sigma_k)(1-\sigma_E)}{(1-\sigma_Q)(1+\sigma)}\left[1 - \exp\frac{-(1-\sigma_E)\tau_2^*}{1+\sigma}\right] -$$
$$- \frac{(1+\sigma)}{(1-\sigma_Q)(1-\sigma_E)} J\left[\frac{(1-\sigma_E)\tau_2^*}{1+\sigma}\right] + \frac{2}{1-\sigma_E} - \left[\frac{2}{1-\sigma_E} + \frac{2\tau_2^*}{1+\sigma} + \frac{(1-\sigma_E)\tau_2^{*2}}{(1+\sigma)^2}\right]\exp\frac{-(1-\sigma_E)\tau_2^*}{1+\sigma}. \tag{6.9}$$

Here the function J is determined as in (4.17).

From the condition (6.6) we find

$$\mu_1 = \frac{(1-\sigma_k)(1+\sigma)}{(1-\sigma_Q)(1-\sigma_E)^2}\left[\frac{(1+\sigma)}{(1-\sigma_Q)}\frac{\pi^2}{6} - 2\right] \ . \tag{6.10}$$

Equations (6.7), (6.9), and (6.10) give approximate expressions for the proper value μ of the function q. However, it is necessary to determine an approximate expression for the function $r(\tau)$ in agreement with all the assumptions introduced during the determination of μ and q in order for these equations to be considered as an approximate solution of the problem (1.8)-(1.11). It was assumed in particular that in the neighborhood of $\tau = 1$ the function $r(\tau)$ is equal to unity with the precision of the small exponential terms. Keeping in mind the explicit form of μ, one can conclude from Eq. (1.8) that the region of significant variation in r is the small neighborhood of the point $\tau = \tau_1° = \sigma_E(1 + \sigma)(1 - \sigma_E)^{-1} - \sigma$. In this region we introduce the variable $\tau_1^* = \beta(\tau_1° - \tau)$ in place of τ, and we will seek solutions in the form of the inner expansions

$$r(\tau_1^*) = f_0^{(1)}(\beta)\, r_0(\tau_1^*) + f_1^{(1)}(\beta)\, r_1(\tau_1^*)$$
$$q(\tau_1)^* = n_0^{(1)}(\beta)\, q_0(\tau_1^*) + n_1^{(1)}(\beta)\, q_1(\tau_1^*). \tag{6.11}$$

The point $\tau = \tau_1°$ is an inner point of the interval, so that the boundary conditions which the functions (6.11) must satisfy represent the conditions of joining the inner expansions (6.11) with the solutions of Eqs. (1.8) and (1.9) in the outer regions. One can confirm the fact that in the outer regions as $\beta \to \infty$ the function $q(\tau)$ is exponentially close to zero, while the function $r(\tau)$ is exponentially close to unity ($\tau > \tau_1°$) and zero ($\tau < \tau_1°$), so that the conditions of joining take the form

$$r_0(\tau_1^*) \to 0, \quad r_1(\tau_1^*) \to 0, \quad q_0(\tau_1^*) \to 0, \quad q_1(\tau_1^*) \to 0 \quad \text{as} \quad \tau_1^* \to \infty \tag{6.12}$$

$$r_0(\tau_1^*) \to 1, r_1(\tau_1^*) \to 0, q_0(\tau_1^*) \to 0, \quad q_1(\tau_1^*) \to 0 \quad \text{as}\ \tau_1^* \to -\infty. \tag{6.13}$$

Having substituted (6.11) into Eqs. (1.8) and (1.9) written in the variable τ_1^*, after estimating the magnitudes with allowance for the explicit form of μ, one can find that

$$q_0(\tau_1^*) = q_1(\tau_1^*) = 0, \qquad f_0^{(1)}(\beta) = 1, \qquad f_1^{(1)}(\beta) = \beta^{-1}. \tag{6.14}$$

In this case the equation for $r_0(\tau_1^*)$ can be written in the form

$$-\frac{(1-\sigma_E)(1-\sigma_Q)}{\sigma_k(1-\sigma_k)(1+\sigma)}\frac{dr_0}{d\tau_1^*} = \frac{1-r_0}{\tau_1° - \sigma_Q r_0}\exp\frac{-\sigma_E(1+\sigma)\tau_1^*}{(\tau_1° + \sigma)^2}. \tag{6.15}$$

From (6.15) and (6.12) we find the implicit expression for $r_0(\tau_1^*)$:

$$(\sigma_Q - \tau_1°)\ln(1 - r_0) + \sigma_Q r_0 = \frac{\sigma_k(1-\sigma_k)(\tau_1° + \sigma)^2}{(1-\sigma_Q)(1-\sigma_E)\sigma_E}\exp\frac{-\sigma_E(1+\sigma)\tau_1^*}{(\tau_1° + \sigma)^2}. \tag{6.16}$$

The function $r_0(\tau_1^*)$ determined by Eq. (6.16) satisfies the joining condition (6.13) only if

$$\tau_1° > \sigma_Q \quad \text{or} \quad \sigma_E > (\sigma_Q + o)(1 + \sigma_Q + 2\sigma)^{-1}. \tag{6.17}$$

The inequality (6.17) gives the lower limit for the region of values of σ_E for which the structure of the wave of the two-stage conversion is described by the solution constructed in this section, i.e., consists of two isolated zones in each of which one of the two successive reactions primarily takes place.

We note that in this case the determination of the function $r_1(\tau_1^*)$ comes down to the solution of an ordinary first-order differential equation not having an analytical solution, and it can be solved through simple numerical integration.

7. Solution for $0 < \sigma_E < (\sigma_Q + \sigma)(1 + \sigma_Q + 2\sigma)^{-1}$. An examination of different variants of the asymptotic behavior of the solution of the problem (1.8)-(1.11) in this range of variation of σ_E leads to the conclusion that the region of the principal variation of r from 0 to 1, as in part 6, turns out to be the small neighborhood of an inner point $\tau = \tau_1°$ outside which the function r differs from 0 and 1 by small exponential terms (for $\tau < \tau_1°$ and $\tau > \tau_1°$, respectively). Now, however, the location of the point $\tau_1°$ does not depend on σ_E and is determined by the equality $\tau_1° = \sigma_Q$.

The behavior of the function q will differ considerably from that in part 6. For $\tau < \sigma_Q$ the function q differs from zero by small exponential terms [for $\tau > \sigma_Q$ the behavior of q is described by the linear function $(\tau - \sigma_Q)(1 - \sigma_Q)^{-1}$ with an accuracy of the small exponential terms].

Let us examine the solution of Eqs. (1.8) and (1.9) in the neighborhood of the point $\tau = \tau_1° = \sigma_Q$. We introduce the variable $\tau_1^* = \beta(\sigma_Q - \tau)$ and construct the solutions in the form of the expansions

$$r(\tau_1^*) = f_0^{(1)}(\beta)\, r_0(\tau_1^*) + f_1^{(1)}(\beta)\, r_1(\tau_1^*)$$
$$q(\tau_1^*) = n_0^{(1)}(\beta) q_0(\tau_1^*) + n_1^{(1)}(\beta)\, q_1(\tau_1^*) \tag{7.1}$$
$$\mu = \varphi_0(\beta)\mu_0 + \varphi_1(\beta)\mu_1.$$

Before writing the equations for the terms of minimum order, we turn our attention to the following circumstance. It follows from the assumption that the principal change of r from 0 to 1 is concentrated near $\tau = \sigma_Q$ that $f_0^{(1)}(\beta) = 1$ in (7.1). In the neighborhood of the point $\tau = \sigma_Q$ where $r \sim 1$ the expansion of the function $q(\tau_1^*)$ cannot begin with a term on the order of unity, since otherwise the condition $\tau \geq \sigma_Q r + (1 - \sigma_Q) q$ will be violated. Consequently, if $n_0^{(1)}(\beta) = 1$, then $q_0(\tau_1^*) = 0$. One other circumstance is connected with the fact that the point $(\sigma_Q, 1, 0)$ is a singular point of Eq. (1.8) (see part 2). The solution of Eq. (1.8) near the point $\tau = \sigma_Q$, where it is assumed the function $q(\tau)$ is smaller than the function $r(\tau)$ by an order of magnitude, must be close to the separatrices, one of which is described by the equations $r = 1$, $q = 0$.

It is seen from Eq. (1.8) that if the function $q(\tau)$ is considerably smaller than the function $r(\tau)$ in the neighborhood of $\tau_1^\circ = \sigma_Q$ so that one can set $q(\tau) = 0$, then in the conversion to the variable $\tau_1^* = \beta(\sigma_Q - \tau)$ the nature of the solution changes greatly since the solution $r(\tau) = 1$ is lost. Therefore, in seeking a function $r(\tau_1^*)$ as a solution in the region $\tau > \sigma_Q$ one must take

$$r_0(\tau_1^*) = 1, \qquad r_1(\tau_1^*) = 0, \qquad \tau_1^* < 0 \ . \tag{7.2}$$

Considering these remarks, for $\tau_1^* < 0$ we arrive at the following equation for the function $r_0(\tau_1^*)$:

$$-\mu_0 \varphi_0(\beta)\beta \frac{dr_0}{d\tau_1^*} = \frac{\sigma_k}{\sigma_Q} \exp\left[-\frac{\beta\sigma_E(1+\sigma)}{\sigma_Q+\sigma} - \frac{\sigma_E(1+\sigma)\tau_1^*}{(\sigma_Q+\sigma)^2}\right] . \tag{7.3}$$

It is seen that it is necessary to take

$$\varphi_0(\beta) = \beta^{-1} \exp \frac{-\sigma_E(1+\sigma)\beta}{\sigma_Q+\sigma} . \tag{7.4}$$

Taking (7.4) into account, we obtain a solution for Eq. (7.3), satisfying the condition of joining with the solution in the outer region $[r_0(\tau_1^*) \to 0,\ \tau_1^* \to \infty]$ in the form

$$r_0(\tau_1^*) = \frac{\sigma_k}{\mu_0 \sigma_Q} \frac{(\sigma_Q+\sigma)^2}{\sigma_E(1+\sigma)} \exp \frac{-\sigma_E(1+\sigma)\tau_1^*}{(\sigma_Q+\sigma)} . \tag{7.5}$$

The constant μ_0 in (7.5) is determined from the requirement of continuity of the function $r_0(\tau_1^*)$ at the point $\tau = \sigma_Q$; we have

$$\mu_0 = \frac{\sigma_k}{\sigma_Q} \frac{(\sigma_Q+\sigma)^2}{\sigma_E(1+\sigma)}, \qquad r_0(\tau_1^*) = \exp \frac{-\sigma_E(1+\sigma)\tau_1^*}{\sigma_Q+\sigma} . \tag{7.6}$$

After substituting (7.1) into (1.9) and considering the results obtained above, it can be established that the equation for $q_1(\tau_1^*)$ takes on the form

$$-\mu_0 n_1^{(1)} \frac{dq_1}{d\tau_1^*} = \frac{(1-\sigma_k)(r_0 - n_1^{(1)} q_1)}{\sigma_Q(1-r_0) - \beta^{-1}\tau_1^* - (1-\sigma_Q) n_1^{(1)} q_1} \exp\left[\delta_q - \frac{\sigma_E(1+\sigma)\tau_1^*}{(\sigma_Q+\sigma)^2}\right],$$
$$\delta_q = \frac{-(1-2\sigma_E)(1+\sigma)}{\sigma_Q+\sigma} < 0. \tag{7.7}$$

In the determination of the function $q(\tau_1^*)$ it is necessary to consider that the function $r_0(\tau_1^*)$ has different forms in the regions $\tau > \sigma_Q$ and $\tau < \sigma_Q$. Then it can be seen from (7.7) that for $\tau < \sigma_Q$, when the function $r_0(\tau_1^*)$ is determined from (7.6), one must set $q_1(\tau_1^*) = 0$ or else choose $n_1^{(1)}(\beta)$ in the form of an exponential function, and not a power function of β^{-1}.

In the region of $\tau > \sigma_Q$, where $r_0(\tau_1^*) = 1$, the function $q_1(\tau_1^*)$ satisfying the condition $q_1(0) = 0$ can be found if one sets $n_1^{(1)}(\beta) = \beta^{-1}$. We obtain

$$q_1(\tau_1^*) = -(1-\sigma_Q)^{-1}\tau_1^* + O(\exp\ \delta_q) . \tag{7.8}$$

The solutions found must be joined with solutions in the neighborhood of the hot boundary $\tau = 1$.

We introduce the variable $\tau^* = \beta(1-\tau)$ into (1.8) and (1.9) in place of τ, and we will seek the functions $r(\tau^*)$ and $q(\tau^*)$ in the form of the expansions

$$r(\tau^*) = f_0(\beta) r_0(\tau^*) + f_1(\beta) r_1(\tau^*)$$
$$q(\tau^*) = n_0(\beta) q_0(\tau^*) + n_1(\beta) q_1(\tau^*). \tag{7.9}$$

By analyzing possible variants of the asymptotic behavior of the functions r and q with (7.4) and the boundary condition (1.11) taken into account, it can be concluded that in the expansion (7.9) one must set $n_0(\beta) = 1$, $f_0(\beta) = 1$. In addition, $r_0(\tau^*) = 1$, $r_1(\tau^*) = 0$, since the difference between $r(\tau^*)$ and unity can be expressed only by components with a much higher order of smallness than the power of β^{-1}.

The equation for $q(\tau^*)$ can be satisfied by setting $n_1(\beta) = \beta^{-1}$. Then

$$q(\tau_1^*) = 1 - \frac{1}{\beta}\frac{\tau^*}{1-\sigma_Q} + O\left(\exp\left[\frac{\sigma_E(1+\sigma_Q+2\sigma)}{\sigma_Q+\sigma} - 1\right]\beta\right). \tag{7.10}$$

The function (7.10) satisfies the boundary condition at the hot boundary. Converting from the variable τ^* to τ_1^* in (7.10), we can confirm that the two-term solution (7.10) joins with the one-term solution (7.8).

The equations obtained exhaust the construction of the approximate asymptotic solution of the problem which makes it possible to determine the zeroth term in the expansion of the proper value.

Changing to the dimensional variable in (7.6), we can write the explicit zeroth approximation of the expression for the mass velocity of the wave of exothermal conversion. In the limiting case under consideration,

$$m^2 = \lambda\rho k_1 \frac{RT_+^{(1)}}{E_1}\frac{T_+^{(1)}}{Q_1}\exp\frac{-E_1}{RT_+^{(1)}}, \quad T_+^{(1)} \equiv T_- + c^{-1}Q_1. \tag{7.11}$$

It is seen that the combustion rate depends on the kinetic characteristics and the adiabatic temperature of the first stage. In this case the zeroth approximation (7.11) coincides with the equation for the propagation rate of a single-stage reaction with the kinetic characteristics k_1 and E_1 and combustion temperature $T_+^{(1)}$ obtained by the method of Zel'dovich and Frank-Kamenetskii [1].

The next term in the expansion of the proper value μ can also be determined. For this it is sufficient to consider the equation for $r_1(\tau_1^*)$ near $\tau = \sigma_Q$ with $\tau < \sigma_Q$. One can obtain

$$\mu_0\frac{dr_1}{d\tau_1^*} = \frac{\sigma_k}{\sigma_Q}\left[\frac{\mu_1}{\mu_0} + \frac{\sigma_E(1+\sigma)\tau_1^{*2}}{(\sigma+\sigma_Q)^3} - \frac{\tau_1^*}{\sigma_Q(1-r_0)}\right]\exp\frac{-\sigma_E(1+\sigma)\tau_1^*}{(\sigma+\sigma_Q)^2}, \quad r_1(0) = 0. \tag{7.12}$$

Integrating (7.12) we find

$$r_1(\tau_1^*) = \frac{\mu_1\sigma_Q\sigma_E(1+\sigma)}{\sigma_k(\sigma+\sigma_Q)^2}\left[1 - \exp\frac{-\sigma_E(1+\sigma)\tau_1^*}{(\sigma+\sigma_Q)^2}\right] + \frac{2(\sigma+\sigma_Q)}{\sigma_E(1+\sigma)} - \left[\frac{\sigma_E(1+\sigma)\tau_1^{*2}}{(\sigma+\sigma_Q)^3} + \frac{2\tau_1^*}{(\sigma+\sigma_Q)} + \frac{2(\sigma+\sigma_Q)}{\sigma_E(1+\sigma)}\right] - \frac{(\sigma+\sigma_Q)^2}{\sigma_E\sigma_Q(1+\sigma)}\cdot J\left[\frac{\sigma_E(1+\sigma)\tau_1^*}{(\sigma+\sigma_Q)^2}\right]. \tag{7.13}$$

From the joining condition $r_1 \to 0$, $\tau_1^* \to \infty$ we obtain

$$\mu_1 = \frac{\sigma_k}{\sigma_Q}\frac{(\sigma+\sigma_Q)^4}{\sigma_E^2(1+\sigma)^2}\left[\frac{\pi^2}{6\sigma_Q} - \frac{2}{\sigma+\sigma_Q}\right]. \tag{7.14}$$

8. Discussion of Results. The analytical equations established in parts 4-7 make it possible to classify the combustion processes according to the given physicochemical characteristics of the condensed system, to approximately calculate the propagation rate of the combustion front, and to study the concentration and temperature profiles. In view of the absence of an exact numerical solution of the problem under consideration let us compare the results obtained with the data of [5] in which detailed numerical calculations were made of the propagation of a combustion wave in a gas, determined by the occurrence of a two-stage consecutive exothermic reaction.

Despite the difference between the propagation of a flame in gas and the combustion of a gasless condensed system, it is not difficult to observe the analogy between the processes distinguished in [5] through an analysis of the results of a numerical calculation and the different asymptotic solutions constructed in the present work.

Having used the apt terminology introduced in [6], the process corresponding to the solution of part 4 must be called convergence, the process of part 6 control, and the process of part 7 separation. It is natural to call the process studied in part 5 incomplete convergence. In the convergence process $E_1 > E_2$, and the combustion rate is primarily determined by the kinetics of the first of the reactions and the adiabatic temperature of complete conversion. In the control process $E_2 > E_1$, $E_1/T_+^{(1)} < E_2/T_+$, and the combustion rate is primarily determined by the kinetics of the second reaction and the adiabatic temperature of complete conversion, while in the separation process $E_2 > E_1$, $E_1/T_+^{(1)} > E_2/T_+$, the combustion rate is determined by the characteristics and adiabatic temperature of the first stage, and the second reaction proceeds by an induction process.

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